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Locally Convex Limit Spaces of Measurable Functions with Order Units and Its Duals

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Abstract—We consider linear normed spaces of measurable functions dominated by positive measurable function powered by real positive parameter. Also, we consider its dual and predual, and we propose a method for constructing a limit spaces of these functional spaces taken by power parameter. We prove that these limit spaces are (LF)-spaces and also prove that the limit spaces presume the relation of duality, i.e., the limit space of predual spaces is predual for the limit space of dominated functions, and the limit space of duals is dual for it. Also, the limit space of predual spaces is embedded into the limit space of dual spaces.

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1. INTRODUCTION

Let (Ω, Σ, μ) be a measurable space of σ-finite measure μ. By $L_0(\Omega, \Sigma, \mu)$ we denote the space of all measurable real-valued functions on Ω. As usual, $L_\infty(\Omega, \Sigma, \mu)$ stands for the space of all almost everywhere bounded functions with the norm defined by the equality

$$||f||_\infty \equiv \inf\{\lambda \in \mathbb{R}^+ | |f(x)| < \lambda \text{ almost everywhere}\}.$$

We follow the notation of [5]. Let $f \in L_0^+(\Omega, \Sigma, \mu)$. We consider the set

$$I(f) = \{g \in L_0(\Omega, \Sigma, \mu) | \exists \lambda \in \mathbb{R}^+ \ -\lambda f \le g \le \lambda f \text{ almost everywhere}\}.$$

Clearly, $I(f)$ is a real vector space. By p_f denote the mapping

$$p_f(g) := \inf\{\lambda \in \mathbb{R}^+ | -\lambda f \le g \le \lambda f \text{ almost everywhere}\}.$$

Evidently, p_f is a norm on $I(f)$ and $I(\mathbf{1}) = L_\infty(\Omega, \Sigma, \mu)$. Such spaces are particular case of the spaces with the order unit. The duals of the spaces with order unit are base norm spaces [1].

Proposition 1 ([5]). *If $f > 0$ almost everywhere, then $u : g \in L_\infty(\Omega, \Sigma, \mu) \mapsto fg \in I(f)$ is an isometrical isomorphism of $L_\infty(\Omega, \Sigma, \mu)$ onto $(I(f), p_f)$.*

Further in this article we assume, that $f > 0$ almost everywhere. Let α be positive real number. By τ_α denote the topology of the norm p_{f^α} in $I(f^\alpha)$.

If $||f||_\infty \le 1$, then $f^\alpha \ge f^\beta$ for $\alpha \le \beta$; therefore, $I(f^\alpha) \supset I(f^\beta)$ for $\alpha \le \beta$. For $\alpha \le \beta$ by τ_α^β we denote the topology induced on $I(f^\beta)$ by topology τ_α, note that $\tau_\alpha^\beta \subset \tau_\beta$ and $I(f^\beta) \subset I(f^\alpha)$.

If $||f^{-1}|| \le 1$ (i.e., $f \ge \mathbf{1}$) the inequality $f^\alpha \le f^\beta$ and the inclusion $I(f^\alpha) \subset I(f^\beta)$ hold for $\alpha \le \beta$. For $\alpha \le \beta$ we define τ_β^α as the topology on $I(f^\alpha)$, which is induced by the topology τ_β, where $\tau_\alpha \supset \tau_\beta^\alpha$.

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